Vector Length (Magnitude) = $\|V\| = \sqrt{(Q_1-P_1)^2+(Q_2-P_2)^2}$ $\overrightarrow{PQ}$

Orthogonal = Right Angle

XY Trace (Z=0)
XZ Trace (Y=0)
YZ Trace (X=0)

$\overrightarrow{PQ} = \langle 3, 2 \rangle$ Component Form

Unit Vector (Magnitude of 1) = Divide by Magnitude

$\langle -2, 5 \rangle$

$(-2)^2 + 5^2 = c^2$

$4 + 25 = c^2$

$\sqrt{29} = c$

$\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$ Unit Vector

Direction = Slope

Standard Position → Starts at 0,0

Standard unit vectors $i = \langle 1, 0 \rangle$, $j = \langle 0, 1 \rangle$

$i \langle 1, 0, 0 \rangle$
$j \langle 0, 1, 0 \rangle$
$k \langle 0, 0, 1 \rangle$

$\vec{V} = \|\vec{V}\| (\cos\theta, \sin\theta)$

Resultant Force = Sum of Vectors

Distance in Space

$C = \sqrt{(V_1)^2 + (V_2)^2 + (V_3)^2}$

(X_1, Y_1, Z_1) / (X_2, Y_2, Z_2)

$\sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2}$

Equation of Sphere (Center at X_0, Y_0, Z_0)

$(X-X_0)^2 + (Y-Y_0)^2 + (Z-Z_0)^2 = r^2$

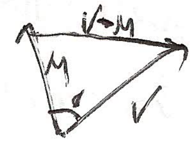
Dot product (Yields a Scalar)

$u\ (4, 1, 6)$

$v\ (-2, 5)$

$u \cdot v = -8 + 30 = 22$ ← Scalar

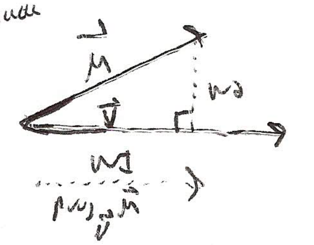
Angle Between Two Vectors



$\cos\theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$

$\frac{\pi}{2}$ = orthogonal

π = parallel

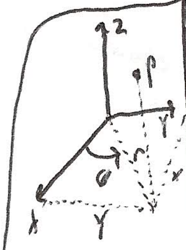


$w_1 = \text{proj}_{\vec{v}} \vec{u}$

w_2 = Vector Component of u that is orthogonal to $\vec{v}$ (↳ Right Angle)

$w_1 + w_2 = u$

$\text{proj}_{\vec{v}} \vec{u} = \left(\frac{u \cdot v}{\|v\|^2} \right) \vec{v}$ ← scalar, ← vector



Rectangular → Cylindrical

$X = r\cos\theta$
$Y = r\sin\theta$
$Z = Z$

Cylindrical to Rectangular

$r^2 = X^2 + Y^2$
$\tan\theta = \frac{Y}{X}$
$Z = Z$

(ρ, θ, ϕ) ← Spherical
(X, Y, Z) ← Rectangular
(r, θ, Z) ← Cylindrical

$Z = 0$ Horizontal plane
$\theta = 0$ Vertical plane

Spherical Coordinates

(ρ, θ, ϕ)

Rho theta phi (fee)

Distance from point to origin

Angle from "North pole" — Positive Z-axis toward it

$\vec{u} \cdot \vec{v} = \langle u_2 \cdot v_3 \rangle$ ...

(i, j, k)

$\vec{u} = \langle u_1, u_2, u_3 \rangle$

$\vec{v} = \langle v_1, v_2, v_3 \rangle$

Cross product $= \vec{u} \times \vec{v} = \langle [(u_2 \cdot v_3) - (u_3 \cdot v_2)], -[(u_1 \cdot v_3) - (u_3 \cdot v_1)], [(u_1 \cdot v_2) - (u_2 \cdot v_1)] \rangle$

Area of parallelogram formed by $\vec{u}$ & $\vec{v}$ is the Magnitude of Cross product ($\|w\|$)

Cross product Magnitude is maximized when degree between two vectors is 90°

Symmetric Equations

$\frac{X-X_1}{a} = \frac{Y-Y_1}{b} = \frac{Z-Z_1}{c}$

Point $(2, 2, 1)$
Vector $\langle -3, 4, 1 \rangle$

$X = 2 - 3t$
$Y = 2 + 4t$
$Z = 1 + 1t$ } Parametric

$\frac{X-2}{-3}, Y = 2, \frac{Z-1}{}$ ← Symmetric Equations

$\frac{X-2}{-3} = Z - 1,\ Y = 2$

Cylindrical → Spherical

$\rho = \sqrt{r^2 + Z^2}$

$\theta = \theta$

$\phi = \arccos\left(\frac{Z}{\sqrt{r^2+Z^2}}\right)$

$r^2 = \rho^2 \sin^2\phi$
$\theta = \theta$
$Z = \rho\cos\phi$

Spherical → Cylindrical

Point (X_1, Y_1, Z_1)
Vector $\langle A, B, C \rangle$

$X = X_1 + at$
$Y = Y_1 + bt$
$Z = Z_1 + ct$

Distance from point to line

$D = \frac{\|\overrightarrow{PQ} \times u\|}{\|u\|}$

At $X = 0$, find point "P", u vector — parallel line

Spherical to Rectangular

$X = \rho\sin\phi\cos\theta$
$Y = \rho\sin\phi\sin\theta$
$Z = \rho\cos\phi$

Rectangular to Spherical

$\rho^2 = X^2 + Y^2 + Z^2$

$\tan\theta = \frac{Y}{X}$

$\phi = \arccos\left(\frac{Z}{\sqrt{X^2+Y^2+Z^2}}\right)$ or $\arccos\left(\frac{Z}{\rho}\right)$

Equation for a Plane: $Ax + By + Cz + D = 0$

Point (X_1, Y_1, Z_1)
Vector $\langle A, B, C \rangle$

$a(X-X_1) + B(Y-Y_1) + C(Z-Z_1) = 0$

Angle between 2 planes in space

$\frac{|n_1 \cdot n_2|}{\|n_1\| \cdot \|n_2\|} = \cos\theta$

Distance Between point & plane

$D = \|\text{proj}_n \overrightarrow{PQ}\| = \frac{|\overrightarrow{PQ} \cdot n|}{\|n\|}$ ← Absolute value

Find any intercept in plane, find $\overrightarrow{PQ}$

$X^2 + Y^2 = 4$

cylinder

CALCULUS 3
EXAM 1 - NOTES

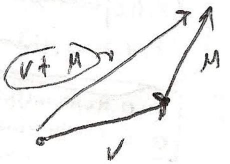
$\vec{M} \cdot \vec{V}$ is orthogonal to both $\vec{M}$ and $\vec{V}$

$||M \cdot V|| = ||M|| \, ||V|| \sin\theta$ = Area of parallelogram having $\vec{M}$ & $\vec{V}$ as adjacent sides

$\vec{M} \cdot \vec{V} = 0$ if and only if $\vec{M}$ & $\vec{V}$ are scalar multiples of each other

Same direction & magnitude = equivalent vectors

Zero vector $\langle 0, 0 \rangle$



Resultant force = sum of vector forces

$\cos\alpha = \frac{v_1}{||v||}$ Angle between v & i

$\cos\beta = \frac{v_2}{||v||}$ Angle between v & j

$\cos\gamma = \frac{v_3}{||v||}$ angle between v & k

If given in i + j + k, then result in i, j, k too!
If given in component form, then result in component form