THE HAPPY/FUN MATH TUTOR

www.HappyFunMathTutor.com

Boca Raton, FL

2901 Clint Moore Road #319, Boca Raton, FL 33496 ♦ Office: (561) 459-2058 ♦ Mobile: (949) 510-8153 ♦ Email: HappyFunMathTutor@gmail.com

www.HappyFunMathTutor.com

GEOMETRY THEORUMS AND POSTULATES

GEOMETRY POSTULATES:

Any segment or angle is congruent to itself. (Reflexive Property)

If there exists a correspondence between the vertices of two triangles such that three sides of one triangle are congruent to the corresponding sides of the other triangle, the two triangles are congruent. (SSS)

If there exists a correspondence between the vertices of two triangles such that two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent. (SAS)

If there exists a correspondence between the vertices of two triangles such that two angles and the included side of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent. (ASA)

Two points determine a line (or ray or segment).

If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other triangle, the two right triangles are congruent. (HL)

A line segment is the shortest path between two points.

Through a point not on a line there is exactly one parallel to the given line. (Parallel Postulate)

Three noncollinear points determine a plane.

If a line intersects a plane not containing it, then the intersection is exactly one point.

If two planes intersect, their intersection is exactly one line.

If there exists a correspondence between the vertices of two triangles such that the three angles of one triangle are congruent to the corresponding angles of the other triangle, then the triangles are similar. (AAA)

A tangent line is perpendicular to the radius drawn to the point of contact.

If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.

Circumference of a circle = $\pi \bullet$ diameter.

The area of a rectangle is equal to the product of the base and the height for that base.

Every closed region has an area.

If two closed figures are congruent, then their areas are equal.

If two closed regions intersect only along a common boundary, then the area of their union is equal to the sum of their individual areas.

The area of a circle is equal to the product of and the square of the radius.

Total area of a sphere = , where r is the sphere's radius.

The volume of a right rectangular prism is equal to the product of its length, its width, and its height.

For any two real numbers x and y, exactly one of the following statements is true x < y, x = y, or x > y. (Law of Trichotomy)

If a > b and b > c, then Q > c. Similarly, if x < y and y < z, then x < z. (Transitive Property of Inequality)

If a > b, then a + x > b + x. (Addition Property of Inequality)

If x < y and a > 0, then a • x < a • y. (Positive Multiplication Property of Inequality)

If x < y and a < 0, then a • x > a • y. (Negative Multiplication Property of Inequality)

The sum of the measures of any two sides of a triangle is always greater than the measure of the third side.

GEOMETRY THEOREMS:

If two angles are right angles, then they are congruent.

If two angles are straight angles, then they are congruent.

If a conditional statement is true, then the contrapositive of the statement is also true. (If p, then q $\Leftrightarrow$ If ~q, then ~p.)

If angles are supplementary to the same angle, then they are congruent.

If angles are supplementary to congruent angles, then they are congruent.

If angles are complementary to the same angle, then they are congruent.

If angles are complementary to congruent angles, then they are congruent.

If a segment is added to two congruent segments, the sums are congruent. (Addition Property)

If an angle is added to two congruent angles, the sums are congruent. (Addition Property)

If congruent segments are added to congruent segments, the sums are congruent. (Addition Property)

If congruent angles are added to congruent angles, the sums are congruent. (Addition Property)

If a segment (or angle) is subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)

If congruent segments (or angles) are subtracted from congruent segments (or angles), the differences are congruent. (Subtraction Property)

If segments (or angles) are congruent, their like multiples are congruent. (Multiplication Property)

If segments (or angles) are congruent, their like divisions are congruent. (Division Property)

If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other. (Transitive Property)

If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other. (Transitive Property)

Vertical angles are congruent.

All radii of a circle are congruent.

If two sides of a triangle are congruent, the angles opposite the sides are congruent.

If two angles of a triangle are congruent, the sides opposite the angles are congruent.

If A = (x_1, y_1) and B = (x_2, y_2), then the midpoint M = (x_m, y_m) of $\overline{AB}$ can be found by using the midpoint formula:

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

If two angles are both supplementary and congruent, then they are right angles.

If two points are each equidistant from the endpoints of a segment, then the two points determine the perpendicular bisector of that segment.

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of that segment.

If two nonvertical lines are parallel, then their slopes are equal.

If the slopes of two nonvertical lines are equal, then the lines are parallel.

If two lines are perpendicular and neither is vertical, each line's slope is the opposite reciprocal of the other's.

If a line's slope is the opposite reciprocal of another line's slope, the two lines are perpendicular.

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

If two lines are cut by a transversal such that two alternate interior angles are congruent, the lines are parallel. (Alt. int. $\angle's \cong \Rightarrow$ || lines)

If two lines are cut by a transversal such that two alternate exterior angles are congruent, the lines are parallel. (Alt. ext. $\angle's \cong \Rightarrow$ || lines)

If two lines are cut by a transversal such that two corresponding angles are congruent, the lines are parallel. (Corr. $\angle's \cong \Rightarrow$ || lines)

If two lines are cut by a transversal such that two interior angles on the same side of the transversal are supplementary, the lines are parallel.

If two lines are cut by a transversal such that two exterior angles on the same side of the transversal are supplementary, the lines are parallel.

If two coplanar lines are perpendicular to a third line, they are parallel.

If two parallel lines are cut by a transversal, each pair of alternate interior angles are congruent. (|| lines $\Rightarrow$ alt. int. $\angle's \cong$)

If two parallel lines are cut by a transversal, then any pair of the angles formed are either congruent or supplementary.

If two parallel lines are cut by a transversal, each pair of alternate exterior angles are congruent. (|| lines $\Rightarrow$ alt. ext. $\angle's \cong$)

If two parallel lines are cut by a transversal, each pair of corresponding angles are congruent. (|| lines $\Rightarrow$ corr. $\angle's \cong$)

If two parallel lines are cut by a transversal, each pair of interior angles on the same side of the transversal are supplementary.

If two parallel lines are cut by a transversal, each pair of exterior angles on the same side of the transversal are supplementary.

In a plane, if a line is perpendicular to one of two parallel lines, it is perpendicular to the other.

If two lines are parallel to a third line, they are parallel to each other. (Transitive Property of Parallel Lines)

A line and a point not on the line determine a plane.

Two intersecting lines determine a plane.

Two parallel lines determine a plane.

If a line is perpendicular to two distinct lines that lie in a plane and that pass through its foot, then it is perpendicular to the plane.

If a plane intersects two parallel planes, the lines of intersection are parallel.

The sum of the measures of the three angles of a triangle is 180°.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

A segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is one-half the length of the third side. (Midline Theorem)

If two angles of one triangle are congruent to two angles of a second triangle, then the third angles are congruent. (No-Choice Theorem)

If there exists a correspondence between the vertices of two triangles such that two angles and a nonincluded side of one are congruent to the corresponding parts of the other, then the triangles are congruent. (AAS)

The sum S_i of the measures of the angles of a polygon with n sides is given by the formula $S_i = (n-2)180$.

If one exterior angle is taken at each vertex, the sum S_e of the measures of the exterior angles of a polygon is given by the formula $S_e = 360$.

The number d of diagonals that can be drawn in a polygon of n sides is given by the formula

$$d = \frac{n(n-3)}{2}$$

The measure E of each exterior angle of an equiangular polygon of n sides is given by the formula

$$E = \frac{360}{n}$$

In a proportion, the product of the means is equal to the product of the extremes. (Means-Extremes Products Theorem)

If the product of a pair of nonzero numbers is equal to the product of another pair of nonzero numbers, then either pair of numbers may be made the extremes, and the other pair the means, of a proportion. (Means-Extremes Ratio Theorem)

The ratio of the perimeters of two similar polygons equals the ratio of any pair of corresponding sides.

If there exists a correspondence between the vertices of two triangles such that two angles of one triangle are congruent to the corresponding angles of the other, then the triangles are similar. (AA~)

If there exists a correspondence between the vertices of two triangles such that the ratios of the measures of corresponding sides are equal, then the triangles are similar. (SSS~)

If there exists a correspondence between the vertices of two triangles such that the ratios of the measures of two pairs of corresponding sides are equal and the included angles are congruent, then the triangles are similar. (SAS~)

If a line is parallel to one side of a triangle and intersects the other two sides, it divides those two sides proportionally. (Side-Splitter Theorem)

If three or more parallel lines are intersected by two transversals, the parallel lines divide the transversals proportionally.

If a ray bisects an angle of a triangle, it divides the opposite side into segments that are proportional to the adjacent sides. (Angle Bisector Theorem)

If an altitude is drawn to the hypotenuse of a right triangle, then:

a. The two triangles formed are similar to the given right triangle and to each other
b. The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse
c. Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)

The square of the measure of the hypotenuse of a right triangle is equal to the sum of the squares of the measures of the legs. (Pythagorean Theorem)

If the square of the measure of one side of a triangle equals the sum of the squares of the measures of the other two sides, then the angle opposite the longest side is a right triangle.

If $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are any two points, then the distance between them can be found with the formula:

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or $PQ = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

In a triangle whose angles have the measures 30º, 60º, and 90º, the lengths of the sides opposite these angles can be represented by x, x$\sqrt{3}$, and 2x respectively. (30°-60°-90"-Triangle Theorem)

In a triangle whose angles have the measures 45º, 45º, and 90º, the lengths of the sides opposite these angles can be represented by x, x, and x$\sqrt{2}$ respectively. (45°-45°-90°-Triangle Theorem)

If a radius is perpendicular to a chord, then it bisects the chord.

If a radius of a circle bisects a chord that is not a diameter, then it is perpendicular to that chord.

The perpendicular bisector of a chord passes through the center of the circle.

If two chords of a circle are equidistant from the center, then they are congruent.

If two chords of a circle are congruent, then they are equidistant from the center of the circle.

If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.

If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.

If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.

If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.

If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.

If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.

If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)

The measure of an inscribed angle or a tangent-chord angle (vertex on a circle) is one-half the measure of its intercepted arc.

The measure of a chord-chord angle is one-half the sum of the measures of the arcs intercepted by the chord-chord angle and its vertical angle.

The measure of a secant-secant angle, a secant-tangent angle, or a tangent-tangent angle (vertex outside a circle) is one-half the difference of the measures of the intercepted arcs.

If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.

If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.

An angle inscribed in a semicircle is a right angle.

The sum of the measures of a tangent-tangent angle and its minor arc is 180°.

If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.

If a parallelogram is inscribed in a circle, it must be a rectangle.

If two chords of a circle intersect inside the circle, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord. (Chord-Chord Power Theorem)

If a tangent segment and a secant segment are drawn from an external point to a circle, then the square of the measure of the tangent segment is equal to the product of the measures of the entire secant segment and its external part. (Tangent-Secant Power Theorem)

If two secant segments are drawn from an external point to a circle, then the product of the measures of one secant segment and its external part is equal to the product of the measures of the other secant segment and its external part. (Secant- Secant Power Theorem)

The length of an arc is equal to the circumference of its circle times the fractional part of the circle determined by the arc.

The area of a square is equal to the square of a side.

The area of a parallelogram is equal to the product of the base and the height.

The area of a triangle is equal to one-half the product of a base and the height (or altitude) for that base.

The area of a trapezoid equals one-half the product of the height and the sum of the bases

The measure of the median of a trapezoid equals the average of the measures of the bases.

The area of a trapezoid is the product of the median and the height.

The area of a kite equals half the product of its diagonals.

The area of an equilateral triangle equals the product of one-fourth the square of a side and the square root of 3.

The area of a regular polygon equals one-half the product of the apothem and the perimeter.

The area of a sector of a circle is equal to the area of the circle times the fractional part of the circle determined by the sector's arc.

If two figures are similar, then the ratio of their areas equals the square of the ratio of corresponding segments. (Similar-Figures Theorem)

A median of a triangle divides the triangle into two triangles with equal areas

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s = semiperimeter = $\frac{a+b+c}{2}$. (Hero's formula)

Area of a cyclic quadrilateral = $\sqrt{s(s-a)(s-b)(s-c)(s-d)}$, where a, b, c, and d are the sides of the quadrilateral and s = semiperimeter = $\frac{a+b+c+d}{2}$. (Brahmagupta's formula)

The lateral area of a cylinder is equal to the product of the height and the circumference of the base.

The lateral area of a cone is equal to one-half the product of the slant height and the circumference of the base.

The volume of a right rectangular prism is equal to the product of the height and the area of the base.

The volume of any prism is equal to the product of the height and the area of the base.

The volume of a cylinder is equal to the product of the height and the area of the base.

The volume of a prism or a cylinder is equal to the product of the figure's cross-sectional area and its height.

The volume of a pyramid is equal to one third of the product of the height and the area of the base.

The volume of a cone is equal to one third of the product of the height and the area of the base.

In a pyramid or a cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figures' respective distances from the vertex.

The volume of a sphere is equal to four thirds of the product of π and the cube of the radius.

The y-form, or slope-intercept form, of the equation of a nonvertical line is $y = mx + b$, where b is the y-intercept of the line and m is the slope of the line.

The formula for an equation of a horizontal line is y = b, where b is the y-coordinate of every point on the line.

The formula for the equation of a vertical line is x = a, where a is the x-coordinate of every point on the line.

If P = (x_1, y_1, z_1) and Q = (x_x, y_2, z_2) are any two points, then the distance between them can be found with the formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The equation of a circle whose center is (h, k) and whose radius is r is

$$(x - h)^2 + (y - k)^2 = r^2$$

The perpendicular bisectors of the sides of a triangle are concurrent at a point that is equidistant from the vertices of the triangle. (The point of concurrency of the perpendicular bisectors is called the circumcenter of the triangle.)

The bisectors of the angles of a triangle are concurrent at a point that is equidistant from the sides of the triangle. (The point of concurrency of the angle bisectors is called the incenter of the triangle.)

The lines containing the altitudes of a triangle are concurrent. (The point of concurrency of the lines containing the altitudes is called the orthocenter of the triangle.)

The medians of a triangle are concurrent at a point that is two thirds of the way from any vertex of the triangle to the midpoint of the opposite side. (The point of concurrency of the medians of a triangle is called the centroid of the triangle.)

If two sides of a triangle are not congruent, then the angles opposite them are not congruent, and the larger angle is opposite the longer side.

If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle.

If two sides of one triangle are congruent to two sides of another triangle and the included angle in the first triangle is greater than the included angle in the second triangle, then the remaining side of the first triangle is greater than the remaining side of the second triangle. $(SAS \neq)$

If two sides of one triangle are congruent to two sides of another triangle and the third side of the first triangle is greater than the third side of the second triangle, then the angle opposite the third side in the first triangle is greater than the angle opposite the third side in the second triangle. $(SSS \neq)$

The distance d from any point P = (x_1, y_1) to a line whose equation is in the form $ax + by + c = 0$ can be found with the formula

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

The area A of a triangle with vertices at (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be found with the formula

$$A = \frac{1}{2}|x_1y_2 + x_2y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2|$$

In any triangle ABC, with side lengths a, b, and c,

$$\frac{a}{\sin \angle A} = D \quad \frac{b}{\sin \angle B} = D \quad \frac{c}{\sin \angle C} = D$$

where D is the diameter of the triangle's circumcircle.

In any triangle ABC, with side lengths a, b, and c,

$$a^2n + b^2m = cd^2 + cmn$$

where d is the length of a segment from vertex C to the opposite side, dividing that side into segments with lengths m and n. (Stewart's Theorem)

If a quadrilateral is inscribable in a circle, the product of the measures of its diagonals is equal to the sum of the products of the measures of the pairs of opposite sides. (Ptolemy's Theorem)

The inradius r of a triangle can be found with the formula $r = \frac{A}{s}$, where A is the triangle's area and s is the triangle's semiperimeter.

The circumradius R of a triangle can be found with the formula $R = \frac{abc}{4A}$, where a, b, and c are the lengths of the sides of the triangle and A is the triangle's area.

lfABCis a triangle with D on $\overline{BC}$, E on $\overline{AC}$, and F on $\overline{AB}$, then the three segments $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are concurrent if, and only if,

$$\left(\frac{BD}{DC}\right)\left(\frac{CE}{EA}\right)\left(\frac{AF}{FB}\right) = 1$$

If ABC is a triangle and F is on $\overline{AB}$, E is on $\overline{AC}$, and D is on an extension of $\overline{BC}$, then the three points D, E, and F are collinear if, and only if,

$$\left(\frac{BD}{DC}\right)\left(\frac{CE}{EA}\right)\left(\frac{AF}{FB}\right) = -1$$