

# Highlights with Formulas



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## Precalculus

Def. The Inverse of a  $2 \times 2$  Matrix  $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  : 
$$\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Def. Determinant of  $2 \times 2$  Matrix: 
$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

or 
$$\det(\mathbf{B}) = |\mathbf{B}| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Def. Determinant of  $3 \times 3$  Matrix  $\mathbf{A} = [a_{ij}] = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$  :

$$|\mathbf{A}| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3 + b_1 c_2 a_3 + a_2 b_3 c_1 - a_3 b_2 c_1 - b_3 c_2 a_1 - a_2 b_1 c_3$$

or "Expanded by Cofactors/Minors": 
$$|\mathbf{A}| = \sum a_{ij} \cdot C_{ij} = \sum a_{ij} (-1)^{i+j} \cdot M_{ij}$$

Def. "Cramer's Rule" (3 Linear Equations in 3 variables).

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} ; \quad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} , \quad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} , \quad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$\rightarrow x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

Def. Determinants and Systems with: 'No-Solutions' or 'Infinitely-Many-Solutions'

I. If  $D = 0$  and At-Least-One of  $D_x, D_y, D_z$  is Not Zero then: System has 'No-Solutions'.

II. If  $D = 0$  and All  $D_x, D_y, D_z$  are Zero then: System has: 'Infinitely-Many-Solutions'  
Or 'No-Solutions'.

Note: (2 Linear Equations in 2 variables):

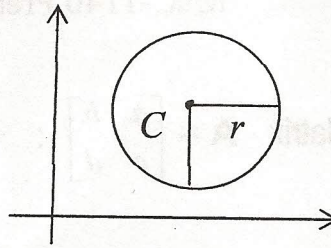
If  $D = 0$  and All  $D_x, D_y$  are Zero then: System has 'Infinitely-Many-Solutions'.

Def. Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Center:  $(h, k)$

$$\frac{(x - h)^2}{r^2} + \frac{(y - k)^2}{r^2} = 1$$



radius:  $r$

Def. Ellipse

A. Horizontal elongation

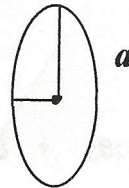
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$



$a > b$

B. Vertical elongation

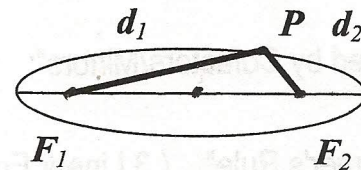
$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$



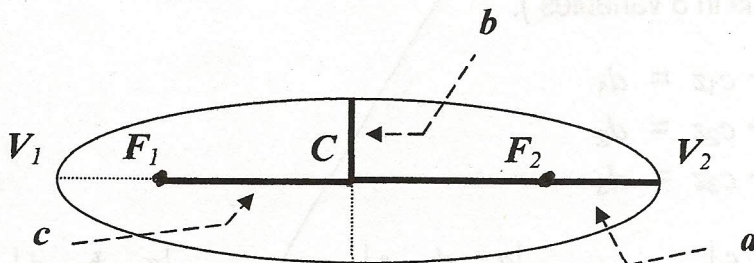
$a > b$

$a, b, c > 0$

Center :  $(h, k)$



$d_1 + d_2 = \text{Constant}$



Vertices (on Major-Axis):  $V_1$  (Vertex # 1),  $V_2$  (Vertex # 2)

$a$  : Distance between Vertex and Center ;  $b$  : Length of Half-of-Minor-Axis-Segment

Foci:  $F_1$  (Focus # 1) ,  $F_2$  (Focus # 1) ;  $c$ : Distance between Focus and Center ,  $c^2 = a^2 - b^2$

Eccentricity:  $e = c/a$



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$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

General II order equation

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

General Conic equation

