PRECALCULUS ROTATED SYSTEMS

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$B^2 - 4AC > 0$ HYPERBOLA
$B^2 - 4AC = 0$ PARABOLA
$B^2 - 4AC < 0$ ELLIPSE

$$\cot 2\phi = \frac{A-C}{B}, \quad 0 < \phi < \frac{\pi}{2}$$

$$\cos\phi = \sqrt{\frac{1+\cos(2\phi)}{2}} \qquad \sin\phi = \sqrt{\frac{1-\cos(2\phi)}{2}}$$

$$\left.\begin{aligned} x &= x'\cos\phi - y'\sin\phi \\ y &= x'\sin\phi + y'\cos\phi \end{aligned}\right\} \quad 0 < \phi < \frac{\pi}{2}$$

$$\left.\begin{aligned} x' &= x\cos\phi + y\sin\phi \\ y' &= -x\sin\phi + y\cos\phi \end{aligned}\right\} \quad 0 < \phi < \frac{\pi}{2}$$

$$\left.\begin{aligned} x &= x' + h \\ y &= y' + k \end{aligned}\right\} \text{TRANSLATION FORMULAS}$$

$$\left.\begin{aligned} x' &= x - h \\ y' &= y - k \end{aligned}\right\}$$

$$A'(x')^2 + B'(x'y') + C'(y')^2 + D'(x') + E'(y') + F' = 0$$

← COEFFICIENTS OF CONICS IN A ROTATED SYSTEM

$$A' = A\cos^2\phi + B\cos\phi\sin\phi + C\sin^2\phi$$

$$B' = B\cos(2\phi) + (C-A)\sin(2\phi)$$

$$C' = C\cos^2\phi - B\cos\phi\sin\phi + A\sin^2\phi$$

$$D' = D\cos\phi + E\sin\phi$$

$$E' = E\cos\phi - D\sin\phi$$

$$F' = F$$

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$2x^2 + \sqrt{3}xy + y^2 - 10 = 0$

$A = 2$

$B = \sqrt{3}$

$C = 1$

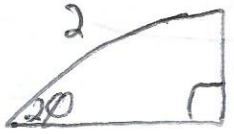
$F = -10$

$D = 0$

$E = 0$

$\cot 2\phi = \frac{A-C}{B}$

$\cot 2\phi = \frac{2-1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$



$(\sqrt{3})^2 + 1^2 = c^2$

$3 + 1 = c^2$

$4 = c^2$

$2 = c$

$\sin 2\phi = \frac{\sqrt{3}}{2}$

$\cos 2\phi = \frac{1}{2}$

$\cos\phi = \sqrt{\frac{1+\cos 2\phi}{2}} = \sqrt{\frac{1+\frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\sin\phi = \sqrt{\frac{1-\cos 2\phi}{2}} = \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$A' = A\cos^2\phi + B\cos\phi\sin\phi + C\sin^2\phi$

$A' = 2\left(\frac{\sqrt{3}}{2}\right)^2 + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}\right)^2$

$A' = 2\left(\frac{3}{4}\right) + \frac{3}{4} + \frac{1}{4} \rightarrow \frac{6}{4} + \frac{3}{4} + \frac{1}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$

$B' = B\cos(2\phi) + (C-A)\sin(2\phi)$

$B' = \sqrt{3}\left(\frac{1}{2}\right) + (1-2)\left(\frac{\sqrt{3}}{2}\right)$

$B' = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$

$C' = C\cos^2\phi - B\cos\phi\sin\phi + A\sin^2\phi$

$C' = 1\left(\frac{\sqrt{3}}{2}\right)^2 - \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)^2$

$C' = \frac{3}{4} - \frac{3}{4} + \frac{1}{2} = \boxed{\frac{1}{2}}$

$D' = D\cos\phi + E\sin\phi$

$D' = 0\left(\frac{\sqrt{3}}{2}\right) + 0\left(\frac{1}{2}\right) = 0$

$E' = E\cos\phi - D\sin\phi$

$0\left(\frac{\sqrt{3}}{2}\right) - 0\left(\frac{1}{2}\right) = 0$

$F' = F \rightarrow F' = -10$

$A'(x')^2 + B'(x'y') + C'(y')^2 + D'(x') + E'(y') + F' = 0$

$\frac{5}{2}(x')^2 + 0(x'y') + \frac{1}{2}(y')^2 + 0(x') + 0(y') - 10 = 0$

$\frac{5}{2}(x')^2 + \frac{1}{2}(y')^2 - 10 = 0$

$5(x')^2 + 1(y')^2 - 20 = 0$

$5(x')^2 + (y')^2 = 20$

$\frac{(x')^2}{4} + \frac{(y')^2}{20} = 1$

ELLIPSE