TRIGONOMETRY VECTORS

TERMINAL POINT $\langle 3, -1, 5\rangle$

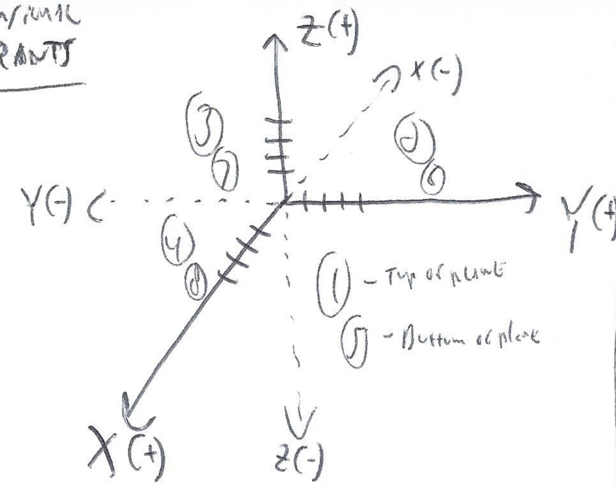
$-$ INITIAL POINT $\langle -1, 3, 2\rangle$

STANDARD VECTOR POSITION $\langle 4, -4, 3\rangle$

$4i - 4j + 3k$

(x, y, z)

3-DIMENSIONAL QUADRANTS



SPHERE

$(x-h)^2 + (y-k)^2 + (z-c)^2 = R^2$

CENTER: (h, k, c)

RADIUS: R

SKETCH THE XZ EQUATION/TRACE

Since "y" is missing, plug in "0" for "y" and solve as regular circle equation

CROSS PRODUCT

$$u = 5i + 3j - k \rightarrow \langle 5, 3, -1\rangle \qquad \|u\| = \sqrt{(5)^2+(3)^2+(-1)^2} = \sqrt{25+9+1} = \sqrt{35}$$

$$v = 2i - j \longrightarrow \langle 2, -1, 0\rangle \qquad \|v\| = \sqrt{(2)^2+(-1)^2+0^2} = \sqrt{4+1+0} = \sqrt{5}$$

ALTERNATE

$$(+)\begin{vmatrix} 3 & -1 \\ -1 & 0 \end{vmatrix} i \; (-) \begin{vmatrix} 5 & -1 \\ 2 & 0 \end{vmatrix} j \; (+) \begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix} k$$

$$[0-(-1)]i + [0-(-2)]j + [(-5)-6]k$$

$\boxed{i + 2j - 11k}$ ← This is orthogonal to both vectors above (i.e., the "dot product" is 0)

Area of parallelogram formed by the adjacent vectors $= \sqrt{(1)^2+(2)^2+(-11)^2} = \sqrt{1+4+121} = \sqrt{126}$ ← Magnitude of cross product

ANGLE BETWEEN VECTORS (3-DIMENSIONAL)

$$\sin\theta = \frac{\|u \times v\|}{\|u\|\,\|v\|} = \frac{\sqrt{126}}{\sqrt{35}\sqrt{5}} = \frac{\sqrt{126}}{\sqrt{175}} = .848528137$$

$\sin\theta = .848528137$

$\theta = 58.05^\circ$

PARALLEL VECTORS:

$-2i + 3j - k \rightarrow \langle -2, 3, -1\rangle$

$4i - 6j + 2k \rightarrow \langle 4, -6, 2\rangle$

$\frac{-2}{4}, \frac{3}{-6}, \frac{-1}{2}$

$-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ ✓

ORTHOGONAL (PERPENDICULAR) VECTORS

Dot product = 0

$-2i + 3j - k \rightarrow \langle -2, 3, -1\rangle$

$3i + 2j \longrightarrow \langle 3, 2, 0\rangle$

$(-2)(3) + (3)(2) + (-1)(0) = -6 + 6 + 0 = 0$ ✓

THREE VECTORS:

$2i + j - k$

$3i - 2j$

$-i + 3j + 2k$

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & 0 \\ -1 & 3 & 2 \end{vmatrix}$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$2\begin{vmatrix} -2 & 0 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 0 \\ -1 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -2 \\ -1 & 3 \end{vmatrix}$$

$$2(-4-0) - 1(6-(0)) + (-1)(9-2)$$

$$-8 - 6 - 7$$

$$\boxed{-21}$$

$|-21| = 21$

↑ VOLUME OF SHAPE FORMED BY THE THREE VECTORS