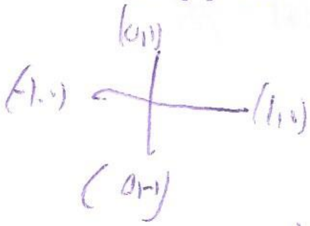
#31

$x^2+4xy+y^2-3=0$ $A=1$, $B=4$, $C=1$

$\cot 2\phi = \frac{A-C}{B}$

$\cot 2\phi = \frac{1-1}{4} = 0$

$\cot 2\phi = 0$ $2\phi = 90°$, $\phi = 45°$




$x = x'\cos 45° - y'\sin 45° \to x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' = \boxed{\frac{\sqrt{2}}{2}(x'-y')}$

$y = x'\sin 45° + y'\cos 45° \to y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \boxed{\frac{\sqrt{2}}{2}(x'+y')}$

$\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2 + 4\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right] + \left[\frac{\sqrt{2}}{2}(x'+y')\right]^2 = 3$

$\frac{1}{2}(x'-y')^2 + 2(x'-y')(x'+y') + \frac{1}{2}(x'+y')^2 = 3$

$(x'-y')^2 + 4(x'-y')(x'+y') + (x'+y')^2 = 6$

$(x')^2 \boxed{\cancel{-2x'y'}} + (y')^2 + 4\left((x')^2-(y')^2\right) + (x')^2 \boxed{\cancel{+2x'y'}} + (y')^2 = 6$

$(x')^2 + (y')^2 + 4(x')^2 - 4(y')^2 + (x')^2 + (y')^2 = 6$

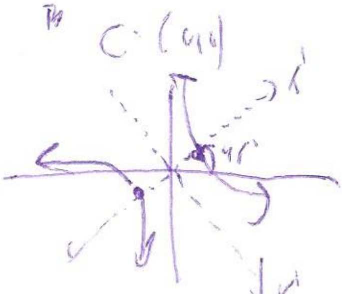
$6(x')^2 - 2(y')^2 = 6$

$\boxed{(x')^2 - \frac{(y')^2}{3} = 1}$

HYPERBOLA

TRANSVERSE AXIS: X'-AXIS (VERTEX ±1,0)

C: (0,0)




## ROTATION OF AXES OF A CONIC SECTION

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$\cot(2\phi) = \frac{A-C}{B}$ $0° < 2\phi < 180°$

$B^2 - 4AC = 0$ (PARABOLA)

$B^2 - 4AC < 0$ (CIRCLE OR ELLIPSE) ← IF COEFFICIENT IN FRONT OF $x^2$ & $y^2$ IS THE SAME, THEN CIRCLE.

$B^2 - 4AC > 0$ (HYPERBOLA)

$x = x'\cos\phi - y'\sin\phi$

$y = x'\sin\phi + y'\cos\phi$

$\sin\phi = \sqrt{\frac{1-\cos(2\phi)}{2}}$

$\cos\phi = \sqrt{\frac{1+\cos(2\phi)}{2}}$

#35

$13x^2 - 6\sqrt{3}xy + 7y^2 - 16 = 0$

$A = 13 \quad B = -6\sqrt{3} \quad C = 7$

$\cot(2\theta) = \frac{A-C}{B} = \frac{13-7}{-6\sqrt{3}} = \frac{6}{-6\sqrt{3}} = \frac{-1}{\sqrt{3}}$ $\qquad \cot = \frac{\cos}{\sin}$

$\sin 2\theta = \frac{\sqrt{3}}{2} \quad \cos 2\theta = \frac{-1}{2}$

USE HALF ANGLE FORMULA + TAN $2\theta$ INTO $\theta$

$\sin\theta = \sqrt{\frac{1-\cos(2\theta)}{2}} = \sqrt{\frac{1-\left(-\frac{1}{2}\right)}{2}} = \sqrt{\frac{\frac{3}{2}}{\frac{2}{1}}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\cos\theta = \sqrt{\frac{1+\cos(2\theta)}{2}} = \sqrt{\frac{1+\left(-\frac{1}{2}\right)}{2}} = \sqrt{\frac{\frac{1}{2}}{\frac{2}{1}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\theta = 60^\circ$ or $\frac{\pi}{3}$

$x = x'\cos 60^\circ - y'\sin 60^\circ = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' = \frac{1}{2}(x' - \sqrt{3}y')$

$y = x'\sin 60^\circ + y'\cos 60^\circ = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' = \frac{1}{2}(\sqrt{3}x' + y')$

$13\left[\frac{1}{2}(x'-\sqrt{3}y')\right]^2 - 6\sqrt{3}\left[\frac{1}{2}(x'-\sqrt{3}y')\right]\left[\frac{1}{2}(\sqrt{3}x'+y')\right] + 7\left[\frac{1}{2}(\sqrt{3}x'+y')\right]^2 = 16$

$\frac{13}{4}(x'-\sqrt{3}y')^2 - \frac{6\sqrt{3}}{4}(x'-\sqrt{3}y')(\sqrt{3}x'+y') + \frac{7}{4}(\sqrt{3}x'+y')^2 = 16$

$13(x'-\sqrt{3}y')^2 - 6\sqrt{3}(x'-\sqrt{3}y')(\sqrt{3}x'+y') + 7(\sqrt{3}x'+y')^2 = 64$

$13\left[(x')^2 - 2\sqrt{3}x'y' + 3(y')^2\right] - 6\sqrt{3}\left[\sqrt{3}(x')^2 + \underbrace{\boxed{x'y' - 3x'y'}}_{-2x'y'} - \sqrt{3}(y')^2\right] + 7\left[3(x')^2 + 2\sqrt{3}x'y' + (y')^2\right] = 64$

$13(x')^2 \cancel{-26\sqrt{3}x'y'} + 39(y')^2 - 18(x')^2 \boxed{\cancel{+12\sqrt{3}x'y'}} + 18(y')^2 + 21(x')^2 \boxed{\cancel{+14\sqrt{3}x'y'}} + 7(y')^2 = 64$

$16(x')^2 - 64(y')^2 = 64$

$\frac{(x')^2}{4} - \frac{(y')^2}{1} = 1$

HYPERBOLA

X-AXIS TRANSVERSE AXIS

$V: (\pm 2, 0)$

$C: (0, 0)$